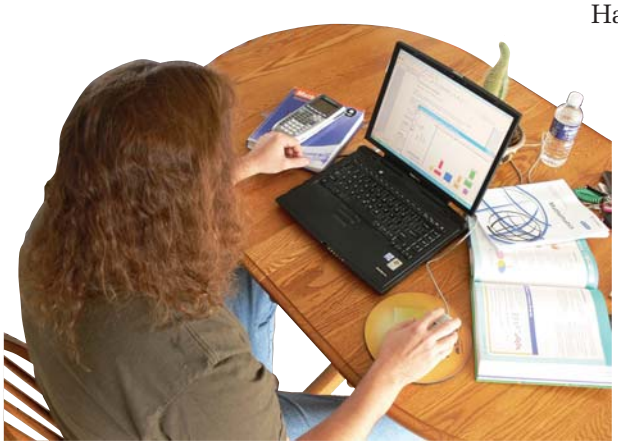
3.6 Add and Subtract Polynomials



Have you ever wondered about who writes textbooks like this one? When do they write these books? How do they get paid?

Experienced mathematics teachers write most math textbooks. They usually write when they are not teaching. They are paid in different ways, depending on their contracts:

- a fixed amount, or flat rate
- a royalty, which depends on the number of books sold
- a combination of fixed amount plus royalty

Why do you think that they are paid in these ways?

Example 1 Add Polynomials

Simplify each expression.

a) $(4x + 3) + (7x + 2)$ **b)** $(2p - 2) + (4p - 7)$

c) $(0.5v^2 + 2v) + (-2.4v^2 - 3v)$

Solution

a) $(4x + 3) + (7x + 2)$ First, remove the brackets, and then collect like terms.

$= 4x + 3 + 7x + 2$

$= 4x + 7x + 3 + 2$

$= 11x + 5$

I can rearrange terms to group like terms.

b) $(2p - 2) + (4p - 7)$

$= 2p - 2 + 4p - 7$

$= 2p + 4p - 2 - 7$

$= 6p - 9$

I use integer rules to collect like terms.

c) Method 1: Remove Brackets and Collect Like Terms

$(0.5v^2 + 2v) + (-2.4v^2 - 3v)$ Apply the integer rules when you remove brackets.

$= 0.5v^2 + 2v - 2.4v^2 - 3v$

$= 0.5v^2 - 2.4v^2 + 2v - 3v$

$= -1.9v^2 - v$

Method 2: Write the Addition Vertically

$$\begin{array}{r} 0.5v^2 + 2v \\ +\ -2.4v^2 - 3v \\ \hline -1.9v^2 - v \end{array}$$

Add the numerical coefficients of the like terms vertically. $0.5 + (-2.4) = -1.9$ and $2 + (-3) = -1$.

$(0.5v^2 + 2v) + (-2.4v^2 - 3v) = -1.9v^2 - v$

Example 2 Opposite Polynomials

State the opposite of each expression.

a) 7 **b)** $-2x$ **c)** $4x + 1$

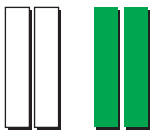
d) $5y - 2$ **e)** $x^2 - 3x + 7$

Solution

Opposites add to give 0.

a) The opposite of 7 is -7.

b) The opposite of $-2x$ is $2x$.

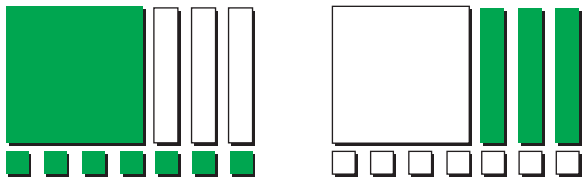


c) To find the opposite of a polynomial, find the opposite of each term.

The opposite of $4x + 1$ is $-4x - 1$.

d) The opposite of $5y - 2$ is $-5y + 2$.

e) The opposite of $x^2 - 3x + 7$ is $-x^2 + 3x - 7$.



Example 3 Subtract Polynomials

Simplify.

a) $(3y + 5) - (7y - 4)$ **b)** $(a^2 - 2a + 1) - (-a^2 - 2a - 5)$

Solution

a) $(3y + 5) - (7y - 4)$

To subtract a polynomial, add its opposite.

$= (3y + 5) + (-7y + 4)$

$= 3y + 5 - 7y + 4$

$= 3y - 7y + 5 + 4$

$= -4y + 9$

The opposite of $7y - 4$ is $-7y + 4$.

b) $(a^2 - 2a + 1) - (-a^2 - 2a - 5)$

$= (a^2 - 2a + 1) + (a^2 + 2a + 5)$

$= a^2 - 2a + 1 + a^2 + 2a + 5$

$= 2a^2 + 6$

$-2a$ and $2a$ are opposites. They add to give 0. There is no a term in the final expression.

Example 4 Apply an Algebraic Model to Solve a Problem

Four authors team up to write a textbook. The publisher negotiates the following contracts with them.

Author	Fixed Rate ($)	Royalty ($ per n books sold)
Lita	2000	–
Jamal	1000	$2n$
Vera	1500	n
Fleming	–	$3n$

a) Write a simplified expression for the total payout to the authors.

b) Determine the total amount paid to the authors for 1200 books sold.

Solution

a) Write an expression for the total amount paid to each author. Then, add these expressions.

Lita: 2000
Jamal: $1000 + 2n$
Vera: $1500 + n$
Fleming: $3n$

Total payout:

$$\begin{aligned} & 2000 + (1000 + 2n) + (1500 + n) + 3n \\ = \ & 2000 + 1000 + 2n + 1500 + n + 3n \\ = \ & 4500 + 6n \end{aligned}$$

The publisher will pay $4500 + 6n$ to the authors, where n is the number of books sold.

b) Substitute $n = 1200$ and evaluate the expression.

$$\begin{aligned} & 4500 + 6n \\ = \ & 4500 + 6(1200) \\ = \ & 4500 + 7200 \\ = \ & 11\ 700 \end{aligned}$$

If 1200 books are sold, the publisher will pay $11 700 to the authors.

Key Concepts

- To add polynomials, remove brackets and collect like terms.
- To subtract a polynomial, add the opposite polynomial.

Communicate Your Understanding

C1 **a)** Describe the process of adding one polynomial to another. Use an example to support your explanation.

b) Describe the process of subtracting one polynomial from another. Use an example to support your explanation.

C2 Each of the following solutions contains an error. Describe the error and suggest how it should be corrected.

a)
$$\begin{aligned}&(2x - 3) + (6x - 2)\\ &= 2x - 3 + 6x - 2\\ &= 8x - 1\end{aligned}$$

b)
$$\begin{aligned}&(4y - 7) - (2y - 5)\\ &= 4y - 7 - 2y - 5\\ &= 2y - 12\end{aligned}$$

Practise

For help with questions 1 and 2, see Example 1.

1. $(2x - 7) + (3x + 8)$ simplified is

A $5x - 15$ **B** $5x - 1$ **C** $5x + 1$ **D** $6x - 56$

2. Simplify by removing brackets and collecting like terms.

a) $(3x + 4) + (7x + 5)$ **b)** $(y + 2) + (3 + 6y)$

c) $(4m - 1) + (3m - 8)$ **d)** $(5 - 3d) + (d - 6)$

e) $(4k - 3) + (5 + k) + (5k + 3)$

f) $(6r - 1) + (3r + 2) + (-6r - 1)$

For help with questions 3 and 4, see Examples 2 and 3.

3. $(3x - 5) - (x - 4)$ simplified is

A $2x - 1$ **B** $2x + 1$ **C** $2x - 9$ **D** $2x + 9$

4. Simplify by adding the opposite polynomial.

a) $(2x + 3) - (x + 6)$ **b)** $(8s + 5) - (s + 5)$

c) $(6m + 4) - (2m + 1)$ **d)** $(4v - 9) - (8 - 3v)$

e) $(9 - 6w) - (-6w - 8)$ **f)** $(5h + 9) - (-5h + 6)$

5. Simplify.

a) $(7x - 9) + (x - 4)$ **b)** $(3y + 8) + (-y - 5)$

c) $(8c - 6) - (c + 7)$ **d)** $(k + 2) - (3k - 2)$

e) $(3p^2 - 8p + 1) + (9p^2 + 4p - 1)$

f) $(5xy^2 + 6x - 7y) - (3xy^2 - 6x + 7y)$

g) $(4x - 3) + (x + 8) - (2x - 5)$

h) $(2uv^2 - 3v) - (v + 3u) + (4uv^2 - 9u)$

Connect and Apply

For help with questions 6 and 7, see Example 4.

6. A group of musicians who made a CD are paid according to the following breakdown, where n is the number of CDs sold.

Musician	Fixed Rate ($)	Royalty ($ for n CDs sold)
Ling	2000	$0.1n$
Fredrick	–	$0.3n$
Nigel	1500	$0.2n$
Tulia	5000	–

a) Find a simplified expression for the total amount paid to the group.

b) The table shows sales achievement levels for the Canadian recording industry.

Status	Number of CDs Sold
Gold	50 000
Platinum	100 000
Diamond	1 000 000

Find the total amount paid to the group if their CD
- sells 100 copies
- reaches gold status
- reaches diamond status

c) Which musician makes the most money at each level in the table in part b)?

d) Describe the advantages and disadvantages of being paid
- by a fixed rate
- by royalty
- by a combination of fixed rate and royalty

Use mathematical reasoning to support your answers.

7. A women's basketball team gives players a bonus of $100 on top of their base salary for every 3-point basket. Data for some of the team's players are given.

Player	Base Salary ($1000s)	3-Point Baskets
Gomez	50	25
Henreid	40	20
Jones	100	44

a) Find a simplified expression for the total earnings for these three players.

b) Find the total earnings for these players.

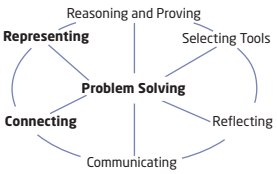
8. Use algebra tiles, virtual algebra tiles, or a diagram to model and simplify each expression.

a) $(2x + 5) + (3x + 2)$

b) $(y^2 + 3y + 1) + (y^2 + 2y + 2)$

c) $(2x^2 + 3x + 4y^2) + (2x^2 + x + 2y^2)$

9. A swimming pool manufacturer installs rectangular pools whose length is twice the width, plus 5 m.



a) Draw a diagram of the pool and label the width and length with algebraic expressions.

b) The entire outer edge of the pool must be fitted with coping, which is a cap used to join the wall of the pool and the deck. Find a simplified algebraic expression that describes the total length of coping needed.

c) How much coping is required if the width of the pool is 6 m?

10. Refer to question 9.

a) Predict how the amount of coping will change if you double the width of the pool.

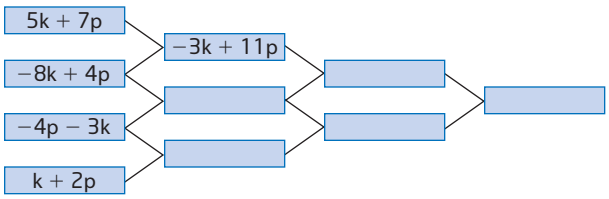
b) Calculate the new amount required and compare this with your prediction. Explain the results.

Extend

11. Refer to question 9. If you use the same expressions for the length and width of the pool as in question 9, do the proportions of the pool change when you change the width? Describe the relationships between width, length, and perimeter as you change the width. Use diagrams, words, and algebraic expressions to support your explanations.

12. **Math Contest**

a) Copy and complete the addition cascade.



b) Build your own addition cascade. Have another student complete it, and then verify each other's work.